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An Extension of the Domain of Influence Theorem for Generalized Thermoelasticity of Anisotropic Material with Voids

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The domain of influence theorem is extended to cover the generalized thermoelasticity of anisotropic bodies with voids in the context of Lord-Shulman and Green-Lindsay theories. We prove that for a finite time $t > 0$ the displacement field u_i, the change in volume fraction ϕ and the temperature T generate no disturbance outside a bounded domain B_t.

Keywords: Influence Theorem, Generalized Thermoelasticity, Anisotropic Material, Voids, Lord-Shulman, Green-Lindsay.

1. INTRODUCTION

It is remarkable to note that the theory of materials with voids or vacuous pores was first proposed by Nunziato and Cowin.[1] In this theory the authors introduce additional degrees of freedom in order to develop the mechanical behavior of a body in which the skeletal material is elastic and interstices are void of material. The intended applications of the theory are to geological materials like rocks and soil and to manufacture porous materials. The linear theory of elastic materials with voids was developed by Cowin and Nunziato in.[2] Here the uniqueness and weak stability of the solutions are also derived. Iesan in[3] has established the equations of thermoelasticity of materials with voids. An extension of these results to cover the theory of dipolar materials was been made in our studies.[4–9] One can find some work on generalized thermoelastic medium with voids in the literature.[10–12] Recently,[13–20] variants problems in waves are studied. Other forms are described for example in the Refs. [21–23].

In the present paper we first consider the basic equations and conditions of the mixed initial-boundary value problem in the context of anisotropic thermoelastic materials with voids. Next we define the domain of influence B_t of the data at time t associated with the problem. We adopt the method used in[24] and[25] to establish a domain of influence theorem. The main result asserts that in the context of theory considered, the solutions of the mixed initial-boundary value problem vanishes outside B_t, for a finite time $t > 0$.

2. BASIC EQUATIONS

An anisotropic elastic material is considered. Assume a such body that occupies a properly regular region B of three-dimensional Euclidian space R^3 bounded by a piecewise smooth surface ∂B and we denote the closure of B by $\bar{B}$. We use a fixed system of rectangular Cartesian axes Ox_i, $(i = 1, 2, 3)$ and adopt the Cartesian tensor notation. A superposed dot stands for the material time derivative while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates. Einstein summation on repeated indices is also used. Also, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion.

The governing equations of the linear theory of anisotropic elastic material with voids and thermal are given by Lord and Shulman,[26] Ciarletta and Scalia,[27] Magana and Quintanilla:[28]

–Constitutive relations

$$t_{ij} = C_{ijkm}e_{km} + D_{ijk}\phi_{,k} + B_{ij}\phi - \beta_{ij}(T + \tau_1\dot{T}) \quad (1)$$

$$h_i = D_{kmi}e_{km} + A_{ij}\phi_{,j} + f_i\phi - a_iT \quad (2)$$

$$g = -B_{ij}e_{ij} - f_i\phi_{,i} - \zeta\phi + b(T + \tau_1\dot{T}) \quad (3)$$

$$\rho\eta = \beta_{ij}e_{ij} + a_i\phi_{,i} + b\phi + aT + a_1\dot{T} \quad (4)$$

$$q_i = K_{ij}T_{,j} \quad (5)$$

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where $(\phi = \nu - \nu_0)$ is the volume fraction field and ν_0 is the matrix volume fraction at the reference state. T is the temperature measured from the absolute temperature $T_0 (T_0 \neq 0)$. We assume that T_0 and ν_0 are positive constants.

–Equations of motion are the balance of linear momentum:

$$\rho \ddot{u}_i = t_{ij,j} + \rho F_i \tag{6}$$

–Balance of equilibrated forces

$$\rho \chi \ddot{\phi} = h_{i,i} + g + \rho l \tag{7}$$

–The energy equation is

$$\rho T_0 \dot{\eta} = -q_{i,i} + \rho r \tag{8}$$

–The generalized Fourier law is

$$q_i + \tau_0 \dot{q}_i = -K_{ij} T_{,j} \tag{9}$$

In the above equations we have used the notations: $C_{ijkm}, D_{ijk}, A_{ij}, B_{ij}, \zeta, f_i, \beta_{ij}, b, a, c, a_i, k_{ij}$ are the constitutive coefficients; ρ is the density; t_{ij} is the symmetric stress tensor; q_i is the heat flux; η is the specific entropy; h_i is the equilibrated stress vector; χ is the equilibrated inertia; F_i is the body force vector; r is the heat supply; g is the intrinsic equilibrated body force and l is the extrinsic equilibrated body force.

If the material symmetry is of a type that posses a center of symmetry then D_{ijk}, a_i and f_i are identically zero.

The general system of equations of motion for anisotropic materials with voids are obtained by the substituting the constitutive relation (1) and (2) into Eqs. (6) and (7)

$$\rho \ddot{u}_i = [C_{ijkm} e_{km} + B_{ij}\phi - \beta_{ij}(T + \tau_1 \dot{T})]_{,j} + \rho F_i \tag{10}$$

$$\rho \chi \ddot{\phi} = -B_{ij} e_{ij} + \left[A_{ij}\phi_{,j}\right]_{,i} - \zeta\phi + b(T + \tau_1 \dot{T}) + \rho l \tag{11}$$

With the help of Eqs. (4), (7) and (9) we get the heat conduction equation for Lord and Shulman theory[9] and from the Eqs (4), (5) and (7) we obtain the heat conduction equation for Green and Lindsay theory[29] and after that we write both these equations in combined form as

$$\rho C^*(\dot{T} + \tau_0 \ddot{T}) + T_0\left(1 + \tau_0 m_0 \frac{\partial}{\partial t}\right)(\beta_{ij}\dot{e}_{ij} + b\dot{\phi}) = K_{ij} T_{,ij} \tag{12}$$

where C^* is the specific heat at the constant strain.

For Lord and Shulman (LS) theory $m_0 = 1$, $\tau_1 = 0$, $a_1 = 0$ and for Green and Lindsay (GL) theory $m_0 = 0$, and $\tau_1 \geq \tau_0 > 0$.

We assume that they are satisfied the following symmetry relations

$$C_{ijmn} = C_{ijnm},\ B_{ij} = B_{ji},\ \beta_{ij} = \beta_{ji},\ K_{ij} = K_{ji} \tag{13}$$

The entropy inequality implies that

$$K_{ij} T_{,i} T_{,j} \geq 0 \tag{14}$$

To the above system of field equations we adjoin the following initial conditions

$$u_i(x,0) = u_i^0(x), \quad \dot{u}_i(x,0) = u_i^1(x), \quad \phi(x,0) = \phi^0(x) \tag{15}$$

$$\dot{\phi}(x,0) = \phi^1(x), \quad T(x,0) = T^0(x), \quad x \in \bar{B} \tag{16}$$

and the following prescribed boundary conditions

$$u_i = \bar{u}_i \text{ on } \partial B_1 \times [0, t_0),$$
$$t_i \equiv t_{ij} n_j = \bar{t}_i \text{ on } \partial B_1^c \times [0, t_0) \tag{17}$$

$$\phi = \bar{\phi} \text{ on } \partial B_2 \times [0, t_0),$$
$$h \equiv h_i n_i = \bar{h} \text{ on } \partial B_2^c \times [0, t_0) \tag{18}$$

$$T = \bar{T} \text{ on } \partial B_3 \times [0, t_0),$$
$$q \equiv q_i n_i = \bar{q} \text{ on } \partial B_3^c \times [0, t_0) \tag{19}$$

where n_i are the components of the unit outward normal to ∂B, t_0 is some instant that may be infinite and $\partial B_1, \partial B_2$ and ∂B_3 with respective complements $\partial B_1^c, \partial B_2^c$ and ∂B_3^c are subsets of ∂B such that

$$\partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c = \partial B_3 \cup \partial B_3^c = \partial B$$
$$\partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c = \partial B_3 \cap \partial B_3^c = \emptyset$$

Also, the above functions $u_i^0, u_i^1, \phi^0, \phi^1, T^0, \bar{u}_i, \bar{\phi}, \bar{h}, \bar{T}$ and $\bar{q}$ are prescribed functions in their domains.

By a solution of the mixed initial boundary value problem of the theory of anisotropic thermoelastic bodies with voids in the cylinder $\Omega_0 = B \times \langle 0, t_0 \rangle$ we mean an ordered array (u_i, ϕ, θ) which satisfies the system of Eqs. (10)–(12) for all $(x, t) \in \Omega_0$, the boundary conditions (17)–(19) and the initial conditions (15) and (16).

3. MAIN RESULTS

We begin this section with the definition of the domain of influence. Next, we establish a domain of influence inequality, which is a counterpart of the inequality established in.[8] Finally, we shall prove a domain influence theorem in the context of anisotropic thermoelastic bodies with voids. In all what follows, we shall use the following assumptions on the material properties

(i) $\rho > 0, I_{ij} > 0, \chi > 0, T_0 > 0, a > 0;$

(ii) $C_{ijmn} x_{ij} x_{mn} + 2D_{ijk} x_{ij} z_k + 2B_{ij} x_{ij}\omega + 2 f_i z_i \omega + \xi\omega^2 + A_{ij} z_i z_j \geq \text{mod}^*4\text{ cm} \geq \alpha(x_{ij}x_{ij} + z_i z_i + \omega^2)$, $\alpha > 0$, for all x_{ij}, z_i, ω;

(iii) $K_{ij}\eta_i\eta_j \geq \gamma\eta_i\eta_i, \gamma > 0$, for all η_i.

These assumptions are in agreement with the usual restrictions imposed in the mechanics of continua. The assumption, (iii) represents a considerable strengthening of the consequence (14) of the entropy production inequality.

For a sufficiently small $\varepsilon > 0$, let $W_\varepsilon(z)$ be a smooth non decreasing function, vanishing in $(-\infty, 0]$ and equal to one in $[\varepsilon, \infty)$, that is

$$W_\varepsilon(z) = \begin{cases} 0, & z \in (-\infty, \varepsilon] \\ 1, & z \in [\varepsilon, \infty) \end{cases}$$

and for $0 \le s \le t$ we define the function $G(x, s)$

$$G(x, s) = W_\varepsilon\left(\frac{R-r}{c} + t - s\right) \tag{20}$$

for some fixed positive R and t, where $r = |x - x_0|$, x_0 is an arbitrary fixed point, c is a positive constant to be determined later.

$G(x, s)$ is a smooth function on $B \times [0, t]$, vanishing outside Σ where

$$\Sigma = \bigcup_{s\in[0,t]} S[x_0, R + c(t-s)]$$

The sphere $S(x_0, R)$ is defined as

$$S(x_0, R) = \{x \in R^3 : |x - x_0| < R\} \tag{21}$$

Let $U(x, s)$ be the function defined as

$$\begin{aligned} U(x, s) = \frac{1}{2}[&\rho\dot{u}_i\dot{u}_i + \rho\chi\dot{\phi}^2 + a(T + \tau_1\dot{T})^2 + C_{ijmn}e_{ij}e_{mn} \\ &+ 2D_{ijk}e_{ij}\phi_{,k} + 2B_{ij}e_{ij}\phi + 2f_i\phi_{,i}\phi \\ &+ A_{ij}\phi_{,i}\phi_{,j} + \zeta\phi^2](x, s) \end{aligned} \tag{22}$$

We also define the function $K(x, s)$

$$K(x, s) = \frac{1}{2}[\rho\dot{u}_i\dot{u}_i + \rho\chi\dot{\phi}^2 + e_{ij}e_{ij} + \phi_{,i}\phi_{,i} + \phi^2](x, s) \tag{23}$$

Taking into account the assumptions (i) and (ii) from (22) and (23) we deduce

$$K(x, s) \le U(x, s) \tag{24}$$

In the next theorem we prove a domain of influence inequality which is a necessary step to prove the main result.

Theorem 1. *Let* (u_i, ϕ, T) *be a solution to the system of Eqs.* (10)–(12) *with the initial conditions* (15), (16) *and the boundary conditions* (17)–(19). *Then for any* $R > 0$, $t > 0$ *and* $x_0 \in B$, *we have that*

$$\begin{aligned} &\int_{D[x_0, R]} U(x, t)\, dV + \frac{1}{T_0}\int_0^t \int_{D[x_0, R+c(t-s)]} K_{ij}T_{,i}T_{,j}\, dV\, ds \\ &\le \int_{D[x_0, R+ct]} U(x, 0)\, dV \\ &+ \int_0^t \int_{D[x_0, R+c(t-s)]} \rho[F_i\dot{u}_i + l\dot{\phi} + \frac{1}{T_0}rT]\, dV\, ds \\ &+ \int_0^t \int_{\partial D[x_0, R+c(t-s)]} [\bar{t}_i\dot{u}_i + \bar{h}\dot{\phi} + \frac{1}{T_0}\bar{q}T]\, dV\, ds \end{aligned} \tag{25}$$

where, we have used the notations

$$D(x_0, R) = \{x \in B : |x - x_0| < R\}$$
$$\partial D(x_0, R) = \{x \in \partial B : |x - x_0| < R\}$$

Proof. Multiplying the Eq. (10) by $G\dot{u}_i$, it results

$$\begin{aligned} \frac{1}{2}G\frac{d}{dt}(\rho\dot{u}_i\dot{u}_i) = \rho GF_i\dot{u}_i + (Gt_{ij}\dot{u}_i)_{,j} - G_{,j}t_{ij}\dot{u}_i \\ - G[C_{ijmn}e_{mn} + D_{ijk}\phi_{,k} \\ + B_{ij}\phi - \beta_{ij}(T + \tau_1\dot{T})]\dot{u}_{i,j} \end{aligned} \tag{26}$$

Multiplying the Eq. (11) by $G\dot{\phi}$, we get

$$\begin{aligned} &\frac{1}{2}G\frac{d}{dt}(\rho\chi\dot{\phi}^2) \\ &= \rho Gl\dot{\phi} + (Gh_i\dot{\phi})_{,i} - G_{,i}h_i\dot{\phi} \\ &\quad - G[D_{ijk}e_{ij} + A_{ij}\phi_{,j} + f_i\phi - a_i(T + \tau_1\dot{T})]\dot{\phi}_{,i} \\ &\quad - G[B_{ij}e_{ij} + f_i\phi_{,i} + \zeta\phi - b(T + \tau_1\dot{T})]\dot{\phi} \end{aligned} \tag{27}$$

At last, multiplying the Eq. (12) by $G(T + \tau_1\dot{T})$, we are led to

$$\begin{aligned} &\frac{1}{2}G\frac{d}{dt}[a(T + \tau_1\dot{T})^2] \\ &= \frac{1}{T_0}Gr\,(T + \tau_1\dot{T}) + \frac{1}{\rho T_0}\{[G\,(T + \tau_1\dot{T})q_i]_{,i} \\ &\qquad - G_{,i}(T + \tau_1\dot{T})q_i\} - \frac{1}{\rho T_0}GK_{ij}T_{,i}T_{,j} \\ &\quad - G[\beta_{ij}\dot{e}_{ij} + a_i\dot{\phi}_{,i} + b\dot{\phi} + a(\dot{T} + \tau_1\ddot{T})](T + \tau_1\dot{T}) \end{aligned} \tag{28}$$

Adding Eqs. (26)–(28) together, it results

$$\begin{aligned} &\frac{1}{2}G\frac{d}{dt}[\rho\dot{u}_i\dot{u}_i + \rho\chi\dot{\phi}^2 + a(T + \tau_1\dot{T})^2] \\ &= \rho GF_i\dot{u}_i + \rho Gl\dot{\phi} + \frac{1}{T_0}GrT + G\left[t_{ij}\dot{u}_i + h_j\dot{\phi} + \frac{1}{\rho T_0}Tq_j\right] \\ &\quad - G[C_{ijmn}e_{mn}\dot{e}_{ij} + D_{ijk}(e_{ij}\dot{\phi}_{,k} + \dot{e}_{ij}\phi_{,k}) \\ &\qquad + B_{ij}(\dot{e}_{ij}\phi + e_{ij}\dot{\phi}) + A_{ij}\phi_{,i}\dot{\phi}_{,j} + f_i(\phi\dot{\phi}_{,i} + \dot{\phi}\phi_{,i}) \\ &\qquad + \zeta\phi\dot{\phi} + a(T + \tau_1\dot{T})(\dot{T} + \tau_1\ddot{T})] - G_{,j}t_{ij}\dot{u}_i \\ &\quad - G_{,i}h_i\dot{\phi} - \frac{1}{\rho T_0}G_{,i}q_iT - \frac{1}{\rho T_0}GK_{ij}\,T_{,i}\,T_{,j} \end{aligned} \tag{29}$$

The relation (29) may be restated as follows

$$\begin{aligned} &\frac{1}{2}G\frac{d}{dt}\{[\rho\dot{u}_i\dot{u}_i + \rho\chi\dot{\phi}^2 + a(T + \tau_1\dot{T})^2] + C_{ijmn}e_{mn}e_{ij} \\ &\quad + 2\,D_{ijk}e_{ij}\phi_{,k} + 2B_{ij}e_{ij}\phi + A_{ij}\phi_{,i}\,\phi_{,j} + 2\,f_i\phi_{,i}\phi + \varsigma\,\phi^2\} \\ &= \rho GF_i\dot{u}_i + \rho Gl\dot{\phi} + \frac{1}{T_0}GrT + G\left(t_{ij}\dot{u}_i + h_j\dot{\phi} + \frac{1}{\rho T_0}Tq_j\right)_{,j} \\ &\quad - G_{,j}t_{ij}\dot{u}_i - G_{,i}h_i\dot{\phi} - \frac{1}{\rho T_0}G_{,i}q_iT - \frac{1}{\rho T_0}GK_{ij}T_{,i}T_{,j} \end{aligned} \tag{30}$$

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It is easy to see that the relation (30) may be written in the form

$$\frac{1}{2}G\dot{U}+\frac{1}{\rho T_0}K_{ij}T_{,i}T_{,j} = G\left(\rho F_i\dot{u}_i+\rho l\dot{\phi}+\frac{1}{T_0}\rho rT\right) + G\left(t_{ij}\dot{u}_i+h_j\dot{\phi}+\frac{1}{\rho T_0}q_jT\right)_{,j} - G_{,j}\left(t_{ij}\dot{u}_i+h_j\dot{\phi}+\frac{1}{\rho T_0}q_jT\right) \quad (31)$$

Integrating both sides of the Eq. (31) over $B\times[0,t]$ and by using the divergence theorem and the boundary conditions (17)–(19), we deduce

$$\int_B GU(x,t)\,dV+\frac{1}{\rho T_0}\int_0^t\int_B GK_{ij}T_{,i}T_{,j}\,dV\,ds = \int_B GU(x,0)\,dV+\int_0^t\int_{\partial B} G\left(\bar{t}_i\dot{u}_i+\bar{h}\dot{\phi}+\frac{1}{\rho T_0}\bar{q}T\right)dA\,ds + \int_0^t\int_B \rho G\left(F_i\dot{u}_i+l\dot{\phi}+\frac{1}{T_0}rT\right)dV\,ds + \int_0^t\int_B \dot{G}U(x,s)\,dV\,ds - \int_0^t\int_B G_{,j}\left(t_{ij}\dot{u}_i+h_j\dot{\phi}+\frac{1}{\rho T_0}q_jT\right)dV\,ds \quad (32)$$

Taking into account the definition (20) of the function G, we find that

$$\left|-G_{,j}t_{ij}\dot{u}_i-G_{,i}h_i\dot{\phi}-\frac{1}{\rho T_0}G_{,i}q_iT\right| = \left|\frac{1}{c}W'_\varepsilon\frac{x_j}{r}(t_{ij}\dot{u}_i+h_j\dot{\phi}+\frac{1}{\rho T_0}q_jT)\right| = \left|\frac{1}{c}W'_\varepsilon\frac{x_j}{r}\left\{[C_{ijmn}e_{mn}+D_{ijk}\phi_{,k}+B_{ij}\phi-\beta_{ij}(T+\tau_1\dot{T})]\dot{u}_i + (D_{jmn}e_{mn}+A_{jk}\phi_{,k}+f_i\phi-a_iT)\dot{\phi} + \frac{1}{\rho T_0}K_{ji}TT_{,i}\right\}\right| \quad (33)$$

Where, we have used the notation

$$W'_\varepsilon=\frac{dW_\varepsilon}{dr}$$

We now make use of arithmetic-geometric mean inequality

$$ab\le\frac{1}{2}\left(\frac{a^2}{p^2}+b^2p^2\right) \quad (34)$$

to the last terms of relation (25) and by choosing suitable parameters p we can find c such that

$$\left|-G_{,j}t_{ij}\dot{u}_i-G_{,i}h_i\dot{\phi}-\frac{1}{T_0}G_{,i}q_iT\right|\le W'_\varepsilon K(x,s) \quad (35)$$

and that

$$\int_0^t\int_B \dot{G}U(x,s)\,dV\,ds - \int_0^t\int_B\left(G_{,j}t_{ij}\dot{u}_i+G_{,i}h_i\dot{\phi}+\frac{1}{T_0}G_{,i}q_iT\right)dV\,ds \le \int_0^t\int_B W'_\varepsilon(x,s)[K(x,s)-U(x,s)]\,dV\,ds\le 0 \quad (36)$$

By using the inequality (36) in Eq. (32), it results

$$\int_B GU(x,t)\,dV+\frac{1}{T_0}\int_0^t\int_B GK_{ij}T_{,i}T_{,j}\,dV\,ds \le\int_B GU(x,0)\,dV+\int_0^t\int_B\rho G(F_i\dot{u}_i+l\dot{\phi}+\frac{1}{\rho^2T_0}rT)\,dV\,ds + \int_0^t\int_{\partial B}G(\bar{t}_i\dot{u}_i+\bar{h}\dot{\phi}+\frac{1}{\rho T_0}\bar{q}T)\,dS\,ds \quad (37)$$

Letting $\varepsilon\to 0$ into relation (37), G tends bounded to the characteristic function of Σ and we get the inequality (25).

Based on the above estimations, we can now prove the main result of our study: the domain of influence theorem.

Let $B(t)$ be the set of points $x\in\bar{B}$ such that:

(1) $x\in B\Rightarrow u_i^0\ne 0\ or u_i^1\ne 0$ or $\phi^0\ne 0\ or \phi^1\ne 0\ or T^0\ne 0$ or $\exists\tau\in[0,t]$ such that $F_i(x,\tau)\ne 0$ or $l(x,\tau)\ne 0$ or $r(x,\tau)\ne 0$;
(2) $x\in\partial B_1\Rightarrow\exists\tau\in[0,t]$ such that $\bar{u}_i(x,\tau)\ne 0$,
(3) $x\in\partial B_1^c\Rightarrow\exists\tau\in[0,t]$ such that $\bar{t}_i(x,\tau)\ne 0$,
(4) $x\in\partial B_2\Rightarrow\exists\tau\in[0,t]$ such that $\bar{\phi}(x,\tau)\ne 0$,
(5) $x\in\partial B_2^c\Rightarrow\exists\tau\in[0,t]$ such that $\bar{h}(x,\tau)\ne 0$,
(6) $x\in\partial B_3\Rightarrow\exists\tau\in[0,t]$ such that $\bar{T}(x,\tau)\ne 0$,
(7) $x\in\partial B_3^c\Rightarrow\exists\tau\in[0,t]$ such that $\bar{q}(x,\tau)\ne 0$.

The domain of influence of the data at instant t is defined as

$$B_t=\{x_0\in\bar{B}\colon B(t)\cap\bar{S}(x_0,ct)\ne\Phi\} \quad (38)$$

where Φ is the empty set.

THEOREM 2. *Let* (u_i,ϕ,T) *be a solution to the system of Eqs.* (10)–(12) *with the initial conditions* (15), (16) *and the boundary conditions* (17)–(19). *Then we have*

$$u_i=0,\phi=0,T=0,\ \ on\ \{\bar{B}\backslash B_t\}\times[0,t]$$

PROOF. For any $x_0\in\bar{B}\backslash B_t$ and $\tau\in[0,t]$, by using the inequality (25) with $t=\tau$ and $R=c(t-\tau)$, we obtain

$$\int_{D[x_0,c(t-\tau)]}U(x,\tau)\,dV+\frac{1}{T_0}\int_0^\tau\int_{D[x_0,c(t-s)]}K_{ij}T_{,i}T_{,j}\,dV\,ds \le\int_{D[x_0,ct)]}U(x,0)\,dV + \int_0^\tau\int_{D[x_0,c(t-s)]}\rho\left(F_i\dot{u}_i+l\dot{\phi}+\frac{1}{T_0}rT\right)dV\,ds + \int_0^\tau\int_{\partial D[x_0,c(t-s)]}\rho\left(\bar{t}_i\dot{u}_i+\bar{h}\dot{\phi}+\frac{1}{T_0}\bar{q}T\right)dS\,ds \quad (39)$$

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Since $x_0 \in \bar{B}\backslash B_t$, we have $x \in D(x_0, ct) \Rightarrow x \notin B(t)$ and hence

$$\int_{D[x_0,\, ct]} U(x, 0)\, dV = 0 \tag{40}$$

Moreover, since $D[x_0,\ c(t-s)] \subseteq D(x_0,\ ct)$, we have

$$\int_0^\tau \int_{D[x_0,\, c(t-s)]} \rho\left(F_i \dot{u}_i + l\dot{\phi} + \frac{1}{T_0} rT\right) dV\, ds = 0 \tag{41}$$

$$\int_0^\tau \int_{D[x_0,\, c(t-s)]} \left(\bar{t}_i \dot{u}_i + \bar{h}\dot{\phi} + \frac{1}{T_0} \bar{q}T\right) dV\, ds = 0 \tag{42}$$

Taking into account the assumption (iii) and the relations (40), (41) and (42) we obtain

$$\int_{D[x_0,\, c(t-\tau)]} U(x, \tau)\, dV \leq 0 \tag{43}$$

Using the inequalities (24) and (43), we get

$$\int_{D[x_0,\, c(t-\tau)]} K(x, \tau)\, dV \leq 0 \tag{44}$$

Taking into account the definition of K and the inequality (44) we deduce

$$\dot{u}_i(x_0, \tau) = 0, \quad \phi(x_0, \tau) = 0, \quad T(x_0, \tau) = 0$$

for any $(x_0, \tau) \in \{\bar{B}\backslash B_t\} \times [0, t]$.

Finally, since $u_i(x_0, 0) = 0$ for any $x_0 \in \bar{B}\backslash B_t$, we deduce

$$u_i(x_0, \tau) = 0, \quad \phi(x_0, \tau) = 0, \quad T(x_0, \tau) = 0$$

for any $(x_0, \tau) \in \{\bar{B}\backslash B_t\} \times [0, t]$ and the proof of Theorem 2 is complete.

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